

Assignment 1

MAST 90023-Simplicial Complexes and Simplicial Maps

Due March 25, 2016

All of these problems are from “Elements of Algebraic Topology” by J. Munkres.

1. Let U be a bounded open set in \mathbb{R}^n . Suppose U is **star-convex** relative to the origin, i.e. that for each x in U , the line segment from $\mathbf{0}$ to x is in U .
 - Show that a ray from $\mathbf{0}$ may intersect $Bd(U)$ in more than one point.
 - Show by example that \bar{U} need not be homeomorphic to \mathbf{B}^n .
2. If $g : |K| \rightarrow |L|$ is a simplicial map carrying the vertices of σ onto the vertices of τ then g maps some face of σ homomorphically onto τ .
3. Let K be a complex. Show that $|K|$ is metrizable if and only if K is locally finite.
4. Let S be a set with a partial order relation \leq . A standard technique in combinatorics is to assign to S the abstract complex \mathcal{S} whose vertices are the elements of S and whose simplices are finite simply-ordered subsets of S . Suppose that S is $\{a_1, \dots, a_8\}$ with partial order generated by the relations:

$$\begin{aligned} a_1 \leq a_3 \leq a_7 \leq a_8; \\ a_1 \leq a_5 \leq a_7; \\ a_2 \leq a_6 \leq a_8; \\ a_2 \leq a_5. \end{aligned} \tag{0.1}$$

5. Show that if G is a finitely generated abelian group, every subgroup of G is finitely generated.

6. Show that if G is free, then G is torsion free.
7. Show that the additive group of the rationals \mathbb{Q} is torsion-free but not free.