UNIVERSITY OF MELBOURNE, SCHOOL OF MATHEMATICS AND STATISTICS

Assignment 2

MAST 90023-Homology

Due April 22, 2016

All of these problems are from “Elements of Algebraic Topology” by J. Munkres.

1. Let $\mathscr{S}$ be the abstract complex consisting of the 1-simplices $\{v_0, v_1\}, \{v_1, v_2\}, \ldots, \{v_n, v_0\}$ and their vertices. If K is a geometric realization of $\mathscr{S}$, compute $H_1(K)$.
2. Define the n-fold connected sum $X_n = T\#T\#\ldots\#T$ of tori, and compute its homology in dimensions 1 and 2.
3. Define the n-fold connected sum $Y_n = P^2\#P^2\#\ldots\#P^2$ of projective planes, and compute its homology in dimensions 1 and 2.
4. Let G be an abelian group and let $\phi : G \to \mathbb{Z}$ be an epimorphism. Show that G has an infinite cyclic subgroup H such that $G = \ker(\phi) \bigoplus H$.
5. Show that if K is a complex and v is a vertex of K, then $H_i(K, v) \cong \tilde{H}_i(K)$ for all i. [Hint: Care is needed when $i = 1$.]
6. Let $f, g : (K, K_0) \to (L, L_0)$ be simplicial maps. Show that if f and g are contiguous as maps of K into L and if L_0 is a full subcomplex of L, then f and g are contiguous as maps of pairs.
7. If $\{C, \epsilon\}$ is an augmented chain complex, show that $\tilde{H}_{-1}(C) = 0$ and
$$\tilde{H}_0(C) \bigoplus \mathbb{Z} \cong H_0(C).$$